IMPLICATIONS OF THE WEIBULL K FACTOR IN RESOURCE ASSESSMENT

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Abstract. This study investigates the implication of the site specific wind speed distribution on the energy production of a wind turbine generator. It will proof that the average wind speed and the power density are not a good measure on how well a turbine will perform; the performance depends on how well a power curve corresponds to the wind speed distribution.

The Weibull distribution is commonly used to describe the probability distribution of wind speed at a given location. Two parameters, the scale factor (c, also sometimes referred to as A) and the shape factor (k) are sufficient to describe a curve which approximates the probability distribution of the wind speed.

The Rayleigh distribution (defined as a Weibull distribution with the shape factor k = 2), is considered frequently as a reference frequency distribution.

EAPC´s practical experience in wind resource assessment in Southern Latin America has revealed a broad variety of shape factors beyond the standard k=2. We have also found that for a given mean wind speed, different shape factors lead to different magnitude of annual energy production on the other. This is especially the case for high wind speeds, where impact on annual energy production can be 15% or higher.

1 THE WEIBULL FUNCTION IN WIND RESOURCE ASSESSMENT

The Weibull distribution is a probability density function, which is used in wind resource assessment to describe wind speed as a stochastic quantity. Typically, in a wind resource measurement campaign, the 10 minute average values of the wind speed are measured, along with other quantities such as wind direction, temperature, pressure, etc. The Weibull function is then used to describe the frequency distribution of the 10 minutes average values over the measurement or evaluation time span (e.g., one year).

The Weibull distribution function is defined as:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}} \qquad (1)$$

where v is the wind speed, k is the shape factor and c is the scale factor. Whilst the scale factor c describes the statistical dispersion of the probability distribution curve, the shape factor k determines the shape of the curve.

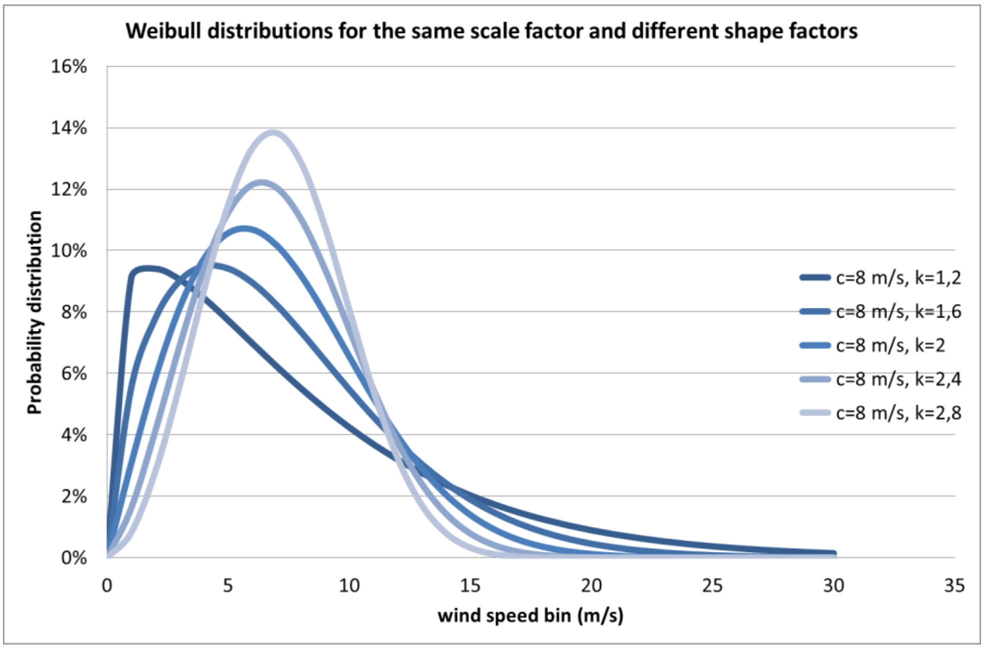

Fig. 1: Weibull distributions for different shape factors at v=8m/s

Typically, the shape factor ranges between 1 and 3. Within this range, for small k values, the Weibull distribution tends towards an exponential distribution, and for high k values the Weibull distribution tends towards a Gaussian distribution. A Weibull distribution with a shape factor of 2 becomes a Rayleigh distribution. The Rayleigh distribution is often referred to as a reference frequency distribution.

With higher k values, the Weibull curve becomes taller and narrower, which in the wind speed range of the investigated wind energy projects 8 – 10 m/s, implies that both very low wind speeds and very high wind speeds are less frequent than at Weibull curves with lower k values. Since the influence of wind speed on the power contained in the wind flow is cubic, especially the shape of the tails of the Weibull curves has impacts on the power density in these environments.

2 THE IMPLICATION OF THE WEIBULL K FACTOR ON POWER DENSITY

Power density is the power contained in the wind flow over the swept rotor area. Referred to the instantaneous speed of a mass unit, the formula for power density is as follows:

$$\frac{P}{A}(v) = \frac{1}{2}\rho v^3 \qquad (2)$$

Where A is the rotor area, ρ is the air density and v is the wind speed.

In order to describe the full power density at a specific location, it is necessary to integrate the power density over the entire wind speed range, multiplied by their probability function (1).

$$\frac{P}{A}(v) = \int_0^\infty \frac{1}{2}\rho v^3 f(v)dv \qquad (3)$$

In the following, two different sites with the same average wind speed of 9m/s but with different shape factors 1,4 and 2,8 are compared. For the sake of comparison, also the air density shall be deemed equal.

Note that the simple application of the formula (2) would lead to a result for the power density of ½ x 1,225 kg/m^3 x (9 m/s)^3 = 447 W/m^2. However, this result underestimates the power density since it ignores the occurrence of higher wind speed values than the mean wind speed.

Applying (3) to either of the two Weibull distributed wind speeds, the results are:

P/A (v=9m/s; k=2,8; c=10,1m/s) = 653 W/m^2

P/A (v=9m/s; k=1,4; c=9,9m/s) = 1.351 W/m^2; an increase of 107%

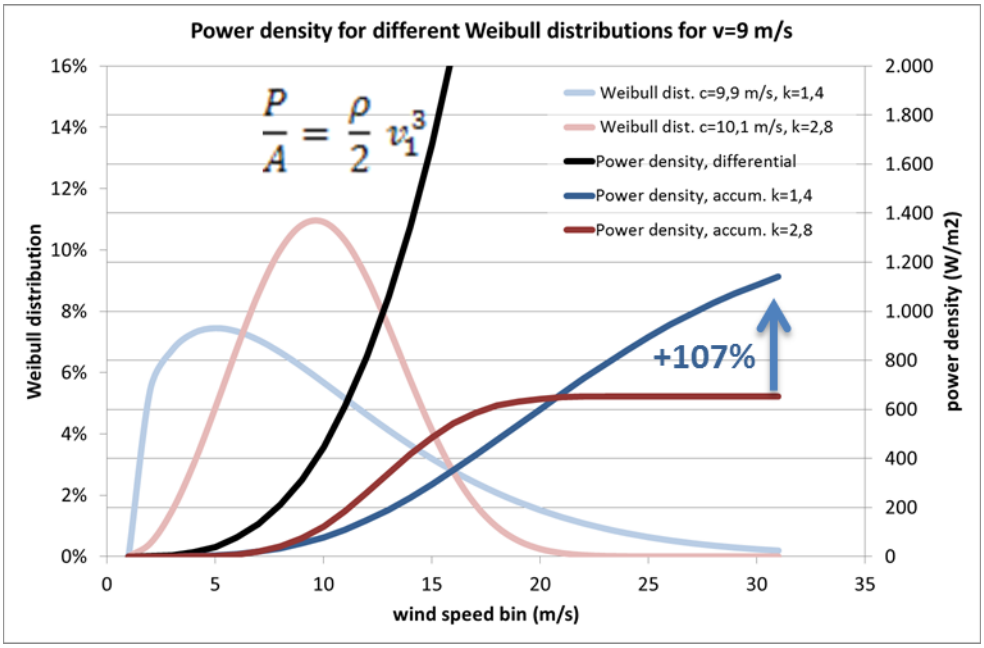

Fig. 2: Power density for different Weibull distributions for v=9m/s

The comparison of the different curves show that with increasing wind speed, although f(v) is decreasing, P/A(v)= ½ x 1,225 kg/m^3 x v^3 is increasing. The Weibull distribution with the shape factor 1,4 has considerably higher occurrences of high wind speed values which is why the corresponding power density curve is still rising, while the Weibull curve with a shape factor of 2,8 is already at its maximum value. Note that the above graph shows accumulated curves for the power density and differential curves for the Weibull distributions.

3 THE IMPLICATION OF THE WEIBULL K FACTOR ON ENERGY YIELD

However, the implication of the Weibull distribution and especially its shape on the power density is not yet relevant for the energy yield assessment of a wind turbine in the specific site environment. The power curve of a wind turbine defines how much of the power in the wind can be effectively extracted by the rotor and converted into electrical energy. The following graph shows typical power curves of class I, II and III wind turbines, being the wind turbine classes identified in IEC 61400-1. It becomes clear that once reaching nominal power, the wind turbine cannot extract any additional power from the wind.

Note that class I wind turbines reach their nominal power at higher wind speeds than class II and III wind turbines, and that their specific nominal power (as related to the rotor area) is higher.

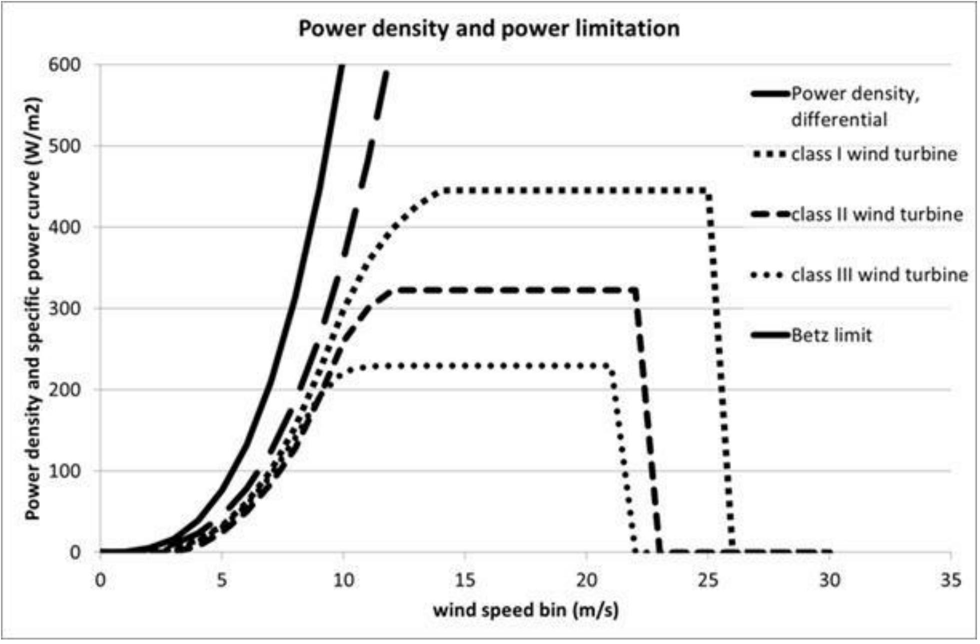

Fig. 3: Power density and power limitation

In a second step, we have investigated the implications of the Weibull function`s shape on the energy production of a wind turbine in high wind regimes (between 7 and 9 m/s). The results are opposite to the implications on power density: the same turbine produces more energy in wind regimes with high shape factors than in wind regimes with low shape factors, given the same mean wind speed and air density.

This is due to the fact that beyond nominal wind speed (which is the speed where the wind turbine reaches nominal power production) the wind turbine cannot extract additional energy from the wind. This is especially disadvantageous at wind regimes with low shape factor, which have high occurrences of very low (below cut-in wind speed) and high (above cut-out wind speed) wind speeds. High shape factor wind speed regimes have generally higher occurrences of wind speed around the nominal wind speed of a wind turbine than low shape factor regimes.

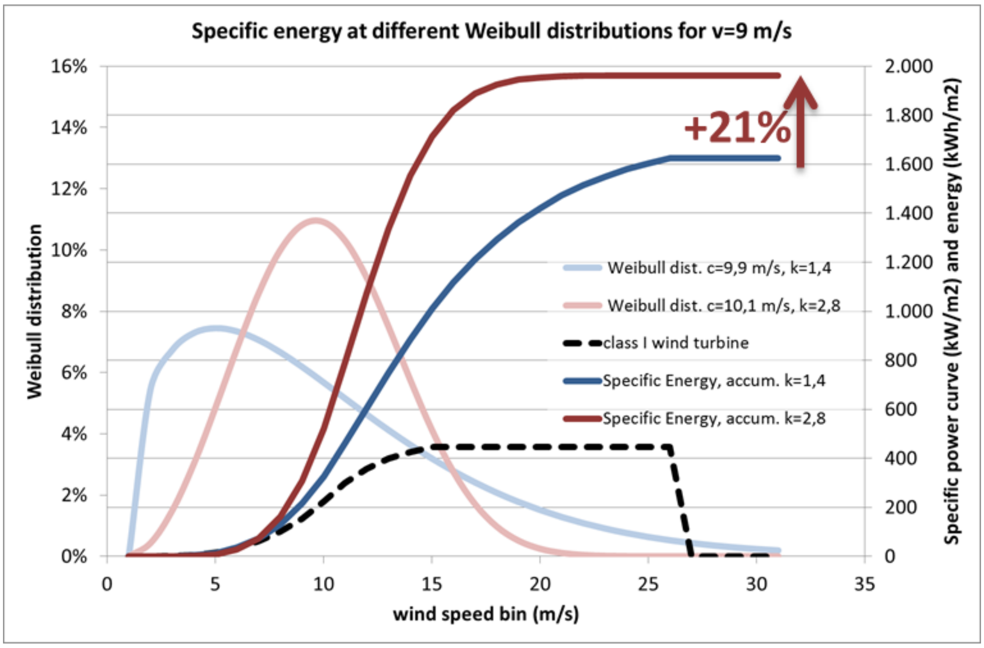

Fig. 4: Specific energy at different Weibull distributions for v=9m/s

The energy production of a typical class I wind turbine at a site with 9 m/s is 1.624 kWh/m2 at a wind regime with k=1,4 and 1.962 kWh/m2 at a wind regime with k=2,8; a difference of 21%. Note that above graph shows accumulated curves for the specific energy and differential curves for the Weibull distributions. This influence decreases with lower wind speeds. Results of similar investigations as described above are shown in Fig. 5.

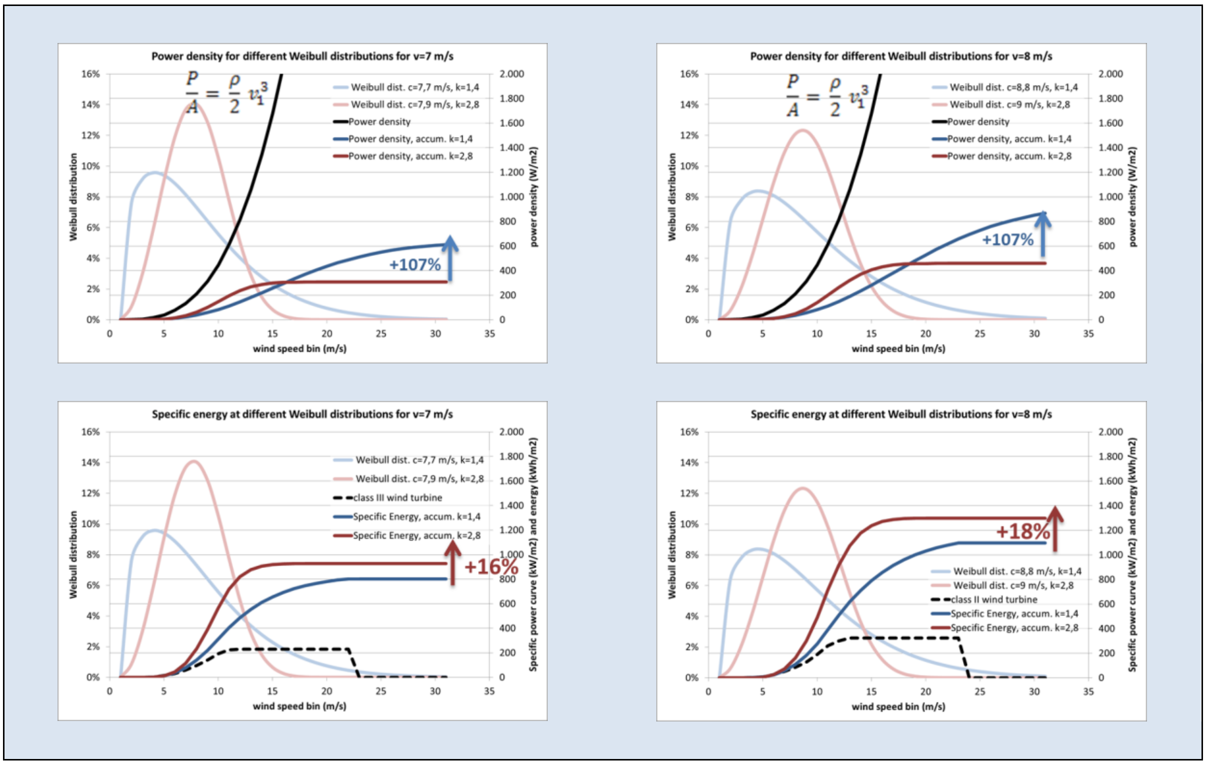

Fig. 5: Specific energy for different wind speeds and different Weibull form factors

4 CONCLUSIONS

The research shows that the Weibull shape has a considerable influence on the energy yield of a wind turbine. Figure 6 shows specific class II wind turbine capacity factors for different shape factors and different mean wind speeds. In the investigated wind speed range from 7 to 9 m/s, high k factors lead to a higher energy yield than low k factors for any given wind speed.

Below 6 – 7 m/s mean wind speed the situation is inverted and low shape factor wind regimes yield higher than high shape factor wind regimes. This threshold value is of course related to the power curve shape and technology dependent.

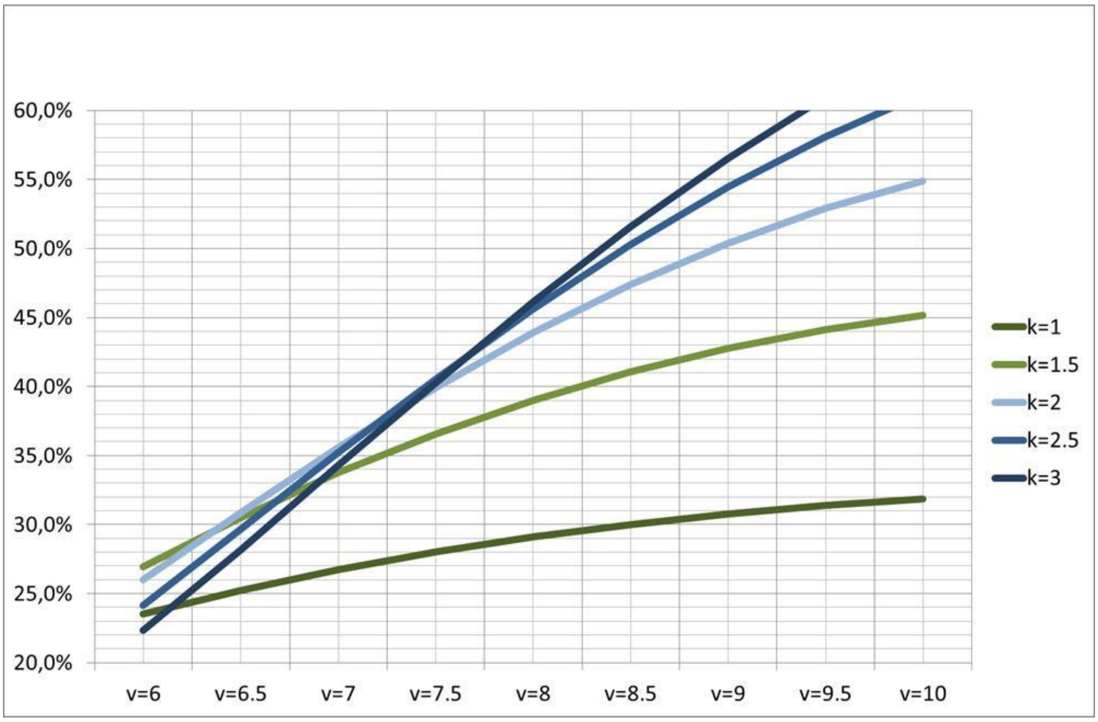

Fig. 6: Capacity factors of a class II wind turbine in different wind regimes characterized by their Weibull k distribution.